Hadrons and Holography

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What is Holographic QCD?

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- Holographic QCD is an attempt to model hadronic physics as a theory of fields or strings in extra dimension(s).
- Simple models of this type capture qualitative, and sometimes quantitative, features of QCD at low energies.

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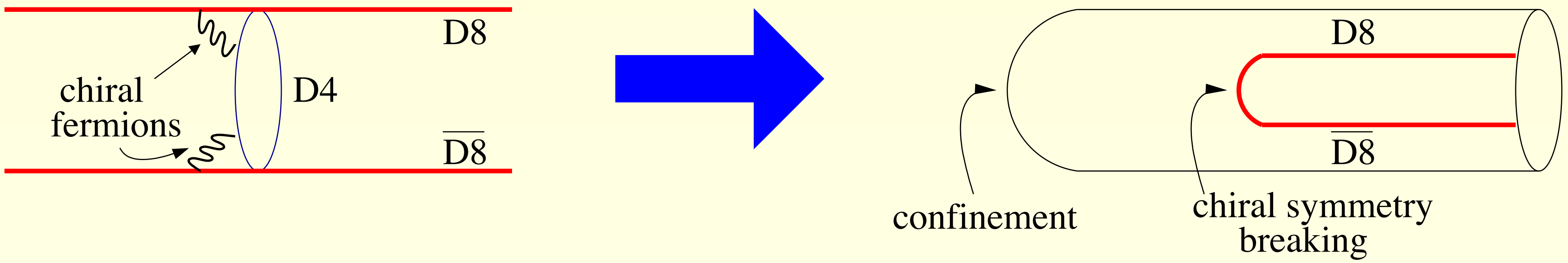
Towers of bound states identified	
	
Turkey	**Tofu**

Top-Down AdS/QCD

- String theory brane configuration → gauge theory similar to QCD (*e.g.* Kruczenski *et al.*; Antonyan,Harvey,Kutasov; Sakai,Sugimoto)
- At large-N, theory has weakly-coupled dual description via the AdS/CFT correspondence (Maldacena)

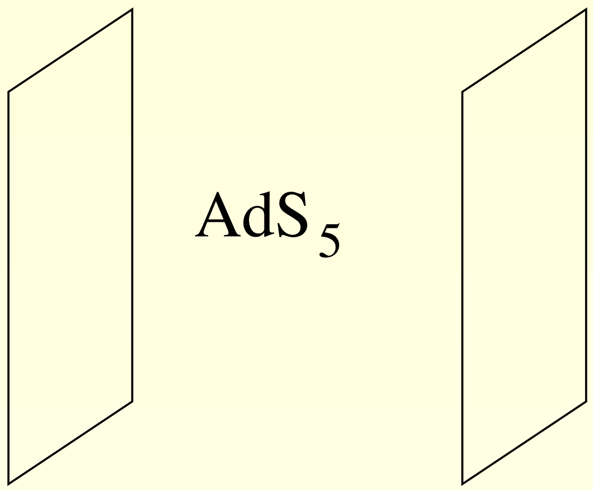
The Sakai-Sugimoto Model



Bottom-Up AdS/QCD

- Model tower of resonances as Kaluza-Klein modes in an extra dimension (Son,Stephanov'04)
- Model pattern of chiral symmetry breaking by analogy with AdS/CFT correspondence

- *Optional*: Specify details of model (geometry of extra dimension, couplings) by matching to UV as best possible (*e.g.* Brodsky,De Teramond; JE *et al.*; Da Rold,Pomarol)



Top-Down AdS/QCD:

- Advantage: Both descriptions of theory are relatively well understood, duality is exact.
- Disadvantage: QCD with fundamental flavors does not have weakly-coupled AdS/CFT dual, so far even at large-N.

Bottom-Up AdS/QCD:

- Advantage: Freedom to match model to aspects of QCD.
- Disadvantage: Some features of model disagree with QCD (analogous to large-N limit).

Building a Bottom-Up Model

Step 1: Choose 5D gauge group and geometry.

- Tower of vector mesons are identified with tower of Kaluza-Klein gauge bosons.

SU(2) isospin → 5D SU(2) gauge theory
Conformal in the UV → Anti-de Sitter space near its boundary

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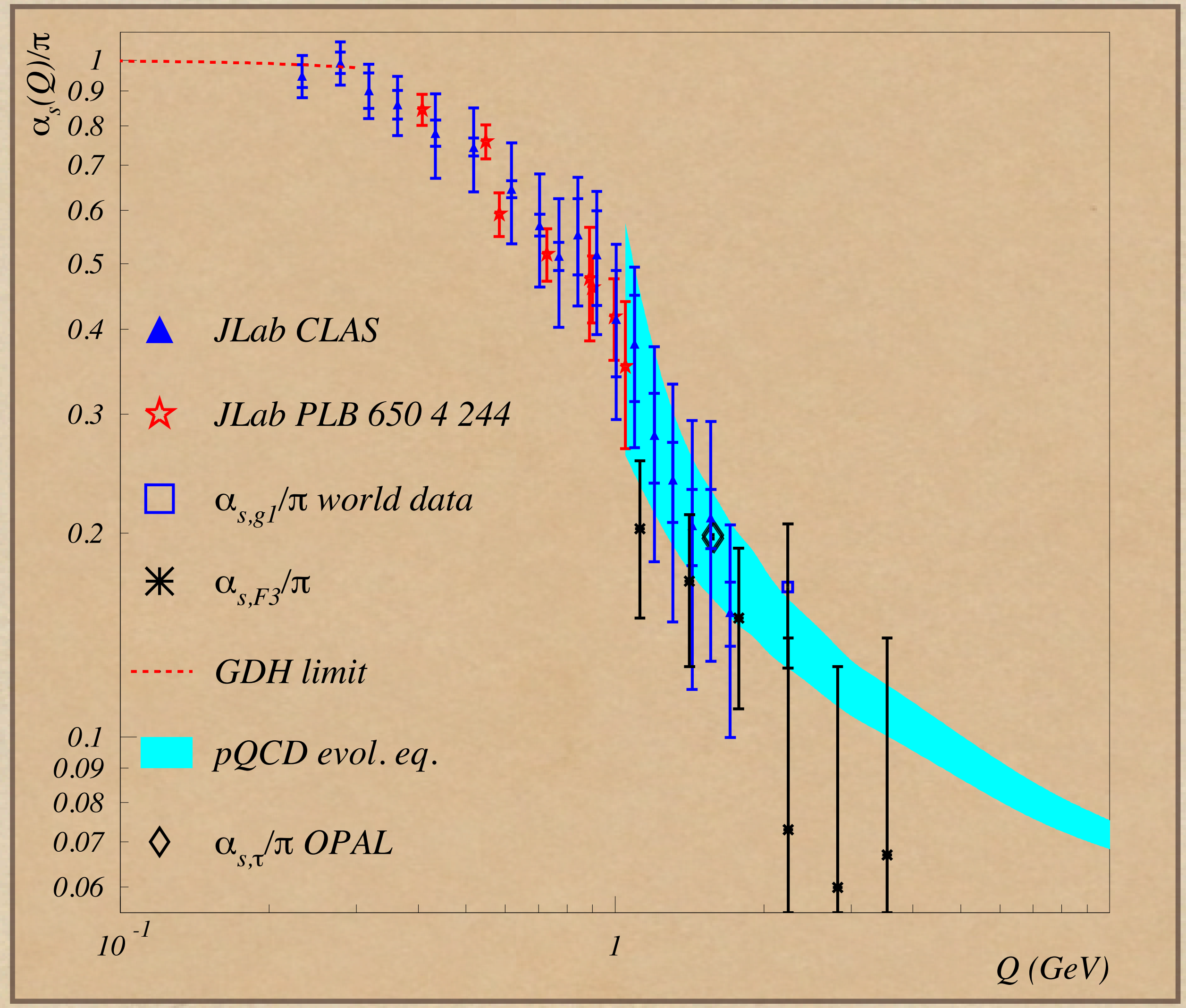
Can choose geometry by matching spectrum to Pade approx of SU(2) current-current correlator in deep Euclidean regime $-q^2 \gg m_\rho^2$.

Result: geometry = slice of AdS space
(Shifman; JE,Kribs,Low; Falkowski,Perez-Victoria).

Evidence for Conformality

Brodsky and collaborators motivate Anti-de Sitter space from approximate conformality of QCD at low energies. e.g. Brodsky and Shrock '08



From CLAS (Deur et al.) '08

Building a Bottom-Up Model

To include the full chiral symmetry, not just the vector subgroup,

SU(2)$\times$SU(2) chiral symmetry $\rightarrow$ SU(2)$\times$SU(2) 5D gauge group

Additional tower of gauge bosons $\rightarrow$ tower of axial-vector mesons.
(5D parity $\rightarrow$ 4D parity)

(Also describes pions after symmetry breaking)

Building a Bottom-Up Model

Step 2: Include pattern of chiral symmetry breaking

Hint from AdS/CFT: 4D operator $\rightarrow$ 5D field
$\overline{q}_i q_j \rightarrow$ Scalar fields X_{ij}, bifundamental under SU(2)×SU(2)

Background profile for X_{ij}:
Non-normalizable mode $\rightarrow$ source $\mathcal{L}_{4D} \supset m_{ij}\, \overline{q}_i q_j$
Normalizable mode $\rightarrow$ VEV $\langle \overline{q}_i q_j \rangle$

The scalar field background explicitly and spontaneously breaks the chiral symmetry.

Building a Bottom-Up Model

For definiteness, we need to choose 5D mass of scalar field.

AdS/CFT: $\boxed{m_X^2 = \Delta_{\bar{q}q}(\Delta_{\bar{q}q} - 4)}$ in units of AdS curvature.

In the UV, $\Delta_{\bar{q}q} = 3$, so we choose $\boxed{m_X^2 = -3}$.

Note: This choice is made for definiteness, but is not necessary.

Building a Bottom-Up Model

In summary, the model is:

SU(2)×SU(2) gauge theory in slice of AdS_5 with background bifundamental scalar field.

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{2g_5^2}\mathrm{Tr}\left(L_{MN}L^{MN} + R_{MN}R^{MN}\right) + \mathrm{Tr}(|D_M X|^2 - 3|X|^2)\right)$$

$$ds^2 = \frac{1}{z^2}\left(dx_\mu dx^\mu - dz^2\right), \quad \epsilon < z < z_{IR}$$

$$X_0(x,z) = \frac{m_q}{2}z + \frac{\langle \overline{q}q\rangle}{2}z^3$$

Model parameters: $g_5,\ m_q,\ \langle \overline{q}q\rangle,\ z_{IR}$

(JE,Katz,Son,Stephanov; DaRold,Pomarol)

Building a Bottom-Up Model

AdS/QCD reproduces consequences of chiral symmetry, *e.g.* Gell-Mann,Oakes,Renner relation

$$m_\pi^2 f_\pi^2 = 2 m_q \langle \overline{q} q \rangle$$

Matching to UV

In the deep Euclidean regime $-q^2 \gg m_\rho^2$, perturbative QCD gives

$$i \int d^4x\, e^{iq\cdot x} \langle J_\mu^a(x) J_\nu^b(0) \rangle = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \delta^{ab} \frac{N}{24\pi^2} \log(q^2)$$

We can express the correlator as a sum over resonances:

$$i \int d^4x\, e^{iq\cdot x} \langle J_\mu^a(x) J_\nu^b(0) \rangle = \sum \frac{F_n^2}{q^2 - m_n^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_n^2}\right) \delta^{ab}$$

Agreement of these expressions in the deep Euclidean regime is a Weinberg sum rule.

$m_n = n^{th}$ Kaluza-Klein mass

$F_n =$ Decay constant of n^{th} resonance

Matching to UV

Matching 5D calculation w/ 4D perturbative calculation in UV →

$$\boxed{g_5^2 = 12\pi^2/N}.$$

Note: this choice is made for definiteness, but is not necessary.

Summary so far:

We have constructed a three-parameter model of the ρ, a_1 and π

Soft-Wall AdS/QCD

In the Hard Wall model $m_n^2 \sim n^2$

To obtain a linear Regge trajectory, the geometry can be modified while coupling to a dilaton background.
(Karch,Katz,Son,Stephanov '06)

$$S = \int d^5x \sqrt{g} e^{-\Phi(x,z)} \mathcal{L}$$

$$\Phi_0(z) \sim z^2, \quad g_{MN} = \mathrm{AdS}_5 \text{ Metric}$$

Low-energy predictions are similar to hard-wall model

Hard-Wall (5D tree level)

With $z_{IR} = 1/(346 \text{ MeV})$, $\langle \bar{q}q \rangle = (308 \text{ MeV})^3$, $m_q = 2.3 \text{ MeV}$

Central Values

Observable	Measured (MeV)	Model (MeV)
m_π	139.6	141
m_ρ	775.8	832
m_{a_1}	1230	1220
f_π	92.4	84.0
$F_\rho^{1/2}$	345	353
$F_{a_1}^{1/2}$	433	440
$g_{\rho\pi\pi}$	6.03	5.29
m_{f_2}	1275	1236

From JE et al. '05, Katz et al. '05

Hard-Wall (5D tree level)

With strange quark mass parameter

Observable	Measured (MeV)	Model (MeV)
m_{K^*}	892	897
m_{ϕ}	1020	994
m_{K_1}	1272	1290
m_K	498	411
f_K	113	117
m_{ω_3}	1667	1656
m_{f_4}	2025	2058
m_{η}	548	520
m'_{η}	958	867

from E. Katz, Lattice 2008

Hard-Wall (5D tree level)

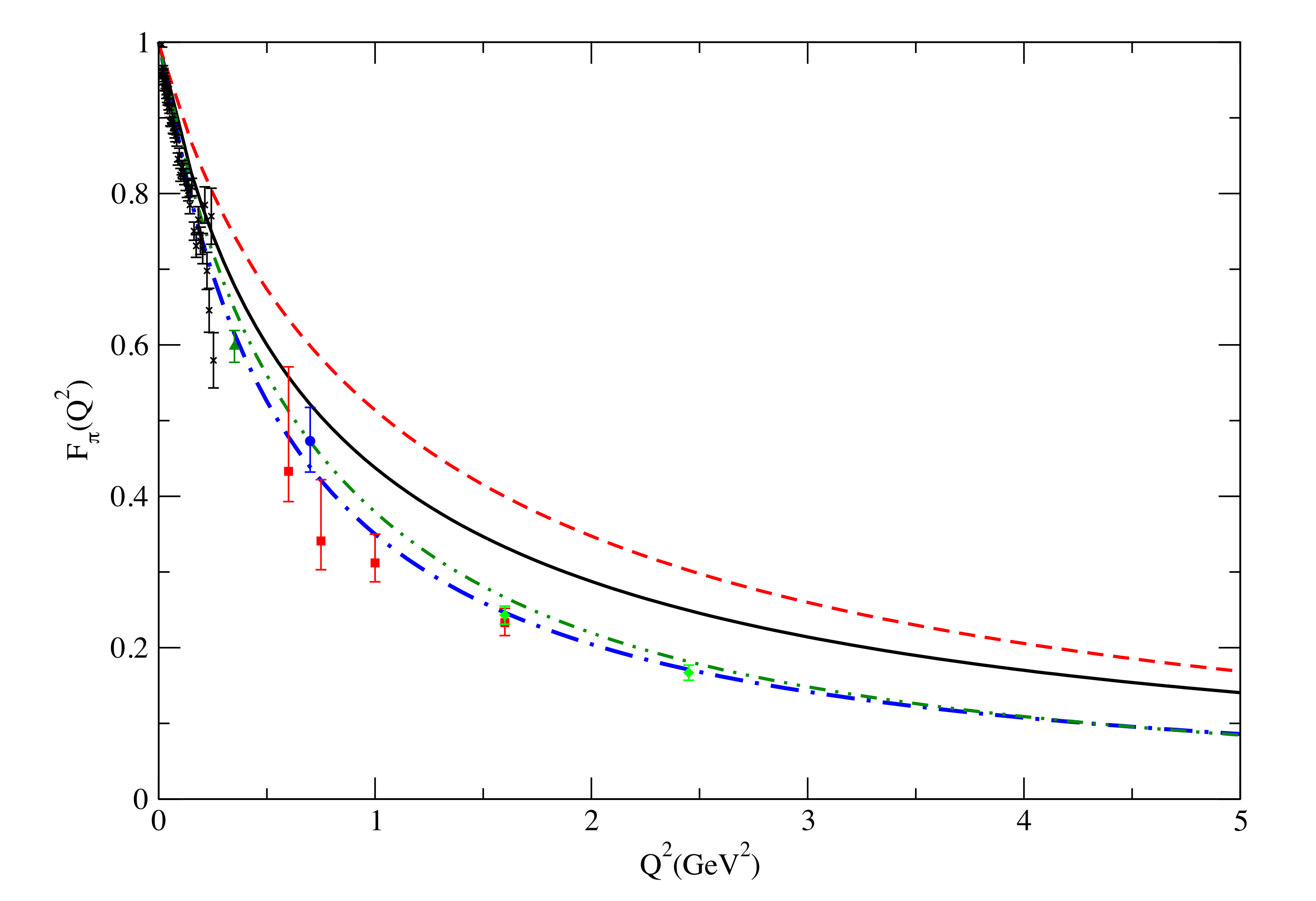
With strange quark mass parameter

Observable	Model A ($\sigma_s = \sigma_q$) (MeV)	Model B ($\sigma_s \neq \sigma_q$) (MeV)	Measured (MeV)
m_π	(fit)	134.3	139.6
f_π	(fit)	86.6	92.4
m_K	(fit)	513.8	495.7
f_K	104	101	113 ± 1.4
$m_{K_0^*}$	791	697	672
$f_{K_0^*}$	28.	36	
m_ρ	(fit)	788.8	775.5
$F_\rho^{1/2}$	329	335	345 ± 8
m_{K^*}	791	821	893.8
$F_{K^*}^{1/2}$	329	337	
m_{a_1}	1366	1267	1230 ± 40
$F_{a_1}^{1/2}$	489	453	433 ± 13
m_{K_1}	1458	1402	1272 ± 7
$F_{K_1}^{1/2}$	511	488	

From Abdidin and Carlson ’09

Hard/Soft-Wall (5D tree level)

Pion Form Factor



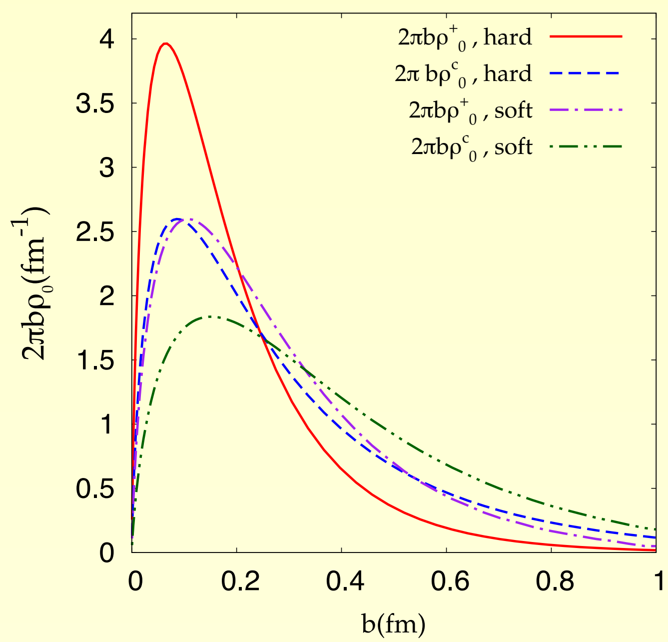
from Kwee and Lebed, arXiv:0807.4565

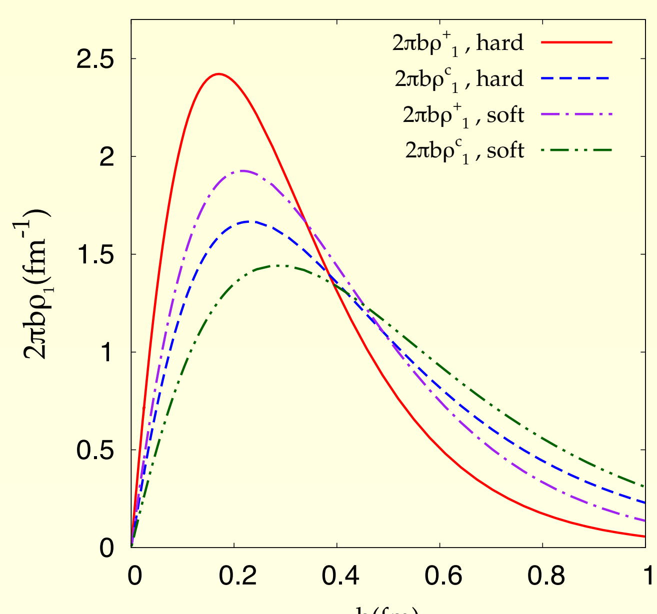
Solid black and blue curves: Hard wall model

Dotted red and green curves: Soft wall model

See also Grigoryan,Radyushkin '08

Hard/Soft-Wall (5D tree level)





Gravitational Form Factors and Generalized Parton Distributions

From Abidin and Carlson, arXiv:0801.3839

Top: p^+ and charge densities of Helicity-0 rho mesons in hard and soft wall models

Bottom: Same for Helicity-1 rho mesons

Hard-Wall (5D tree level)

Can determine meson radii from behavior of form factors near $q^2 = 0$.

Hard wall model:

$$\langle r_\pi^2 \rangle_{charge} = 0.33 \text{ fm}^2$$

$$\langle r_\pi^2 \rangle_{grav} = 0.13 \text{ fm}^2$$

$$\langle r_\rho^2 \rangle_{charge} = 0.53 \text{ fm}^2$$

$$\langle r_\rho^2 \rangle_{grav} = 0.21 \text{ fm}^2$$

$$\langle r_{a_1}^2 \rangle_{charge} = 0.39 \text{ fm}^2$$

$$\langle r_{a_1}^2 \rangle_{grav} = 0.15 \text{ fm}^2$$

H. Grigoryan and A. Radyushkin; Z. Abidin and C. Carlson '07,'08

Universality in AdS/QCD?

Some observables are truly universal, *i.e.* independent of details of model.

Famous Example: Viscosity to Entropy Density η/s

Finite temperature $\rightarrow$ spacetime horizon

Prediction, independent of details of spacetime geometry:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun, Son and Starinets '04

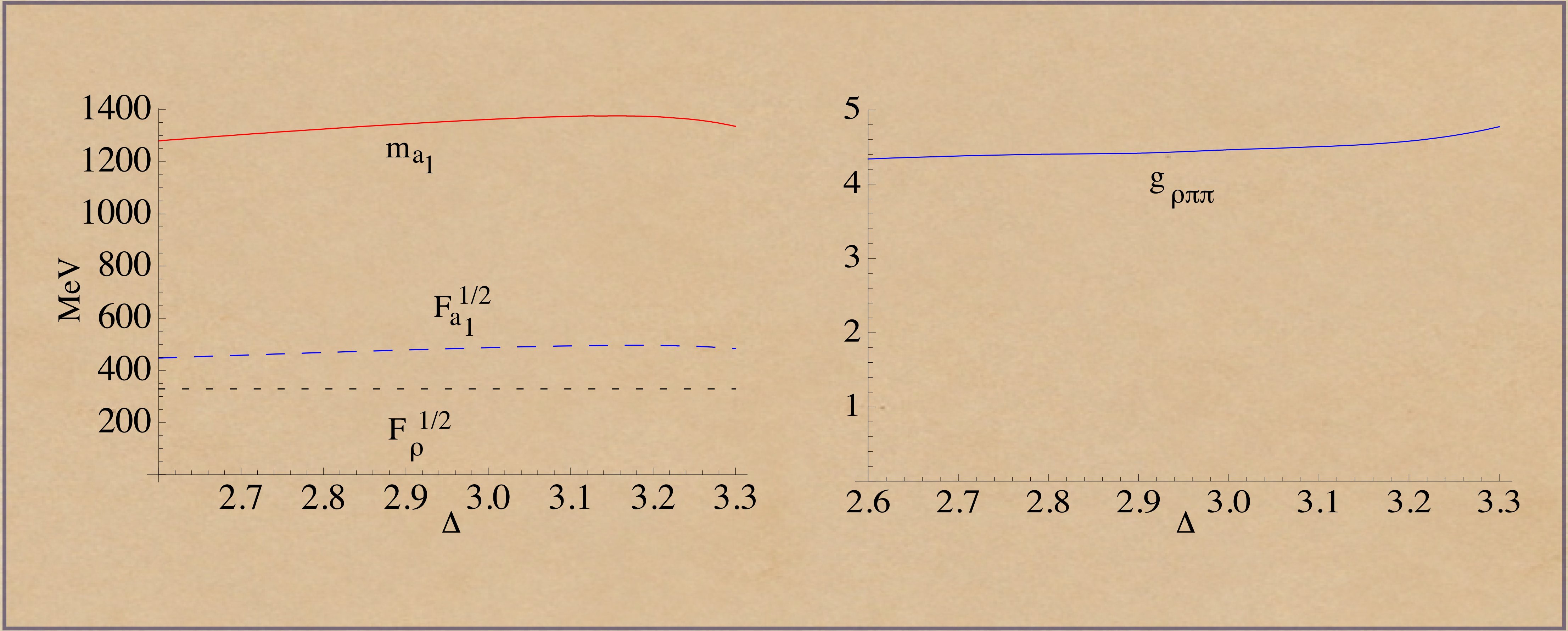
Another example: Electrical Conductivity to Charge Susceptibility σ/χ

$$\frac{\sigma}{\chi} = \frac{1}{4\pi T}\frac{d}{d-2}$$

Kovtun and Ritz '08

Universality?

Observables are roughly independent of X mass



From JE, Westenberger '09

$m_X^2 = \Delta(\Delta - 4)$.

$\Delta = 3$ is the value set by matching to the UV.

Other Observations

- Close relationship between meson wavefunctions in extra dimension and light-front wavefunctions (Brodsky and De Teramond)
- Baryons appear as solitons in extra dimension similar to Skyrmions (Sakai and Sugimoto; Nawa *et al.*; Pomarol and Wulzer)
- AdS/QCD may address qualitative questions like chiral symmetry restoration (D.K. Hong *et al.*; Shifman and Vainshtein; Klempt)
- Can improve matching to UV by adding higher dimension 5D operators to action $\rightarrow$ power corrections in Operator Product Expansion (Hirn and Sanz)

AdS/CFT models have also been used to study the phase structure of QCD; cold atoms and superconductivity (Son;Balsubramanian et al.; Hartnoll et al.); technicolor (Hirn,Sanz;Carone et al.; Hong et al.)

Summary

- Holographic QCD models combine features of other models of QCD at low energies
- 5D tree-level calculations in holographic QCD models agree with hadronic observables at the 10-20% level, sometimes better
- 5D loop corrections and higher-dimension 5D operators have not been included -- AdS/QCD is an uncontrolled approximation above the QCD scale, so its success is a bit surprising.